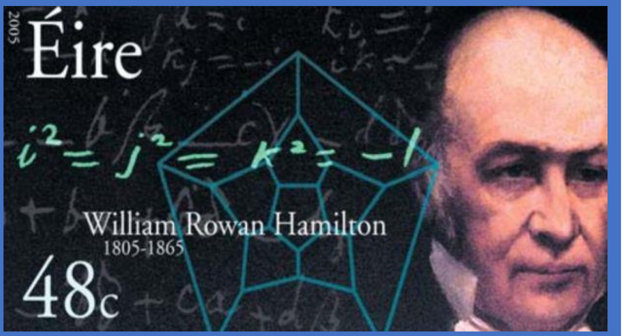

MATHEMATICIANS IN OUR LIVES

With the support of

11-12 years olds

SECTION 1 - WILLIAM ROWAN HAMILTON

1. Who is William Rowan Hamilton?

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2. What was his best known discovery?

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3. Where was the bridge that Hamilton carved equations into?

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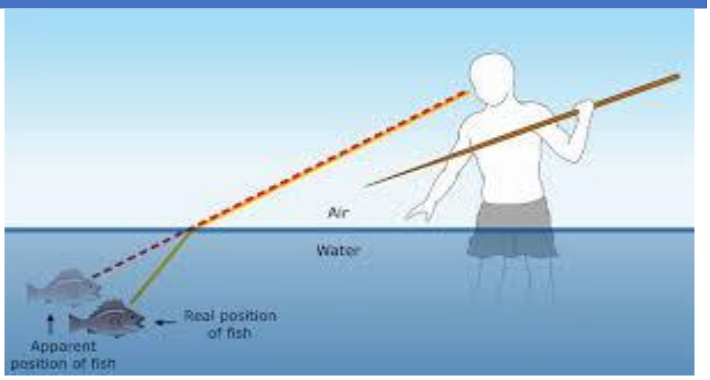
SECTION 2 - OPTICS

4. What is refraction?



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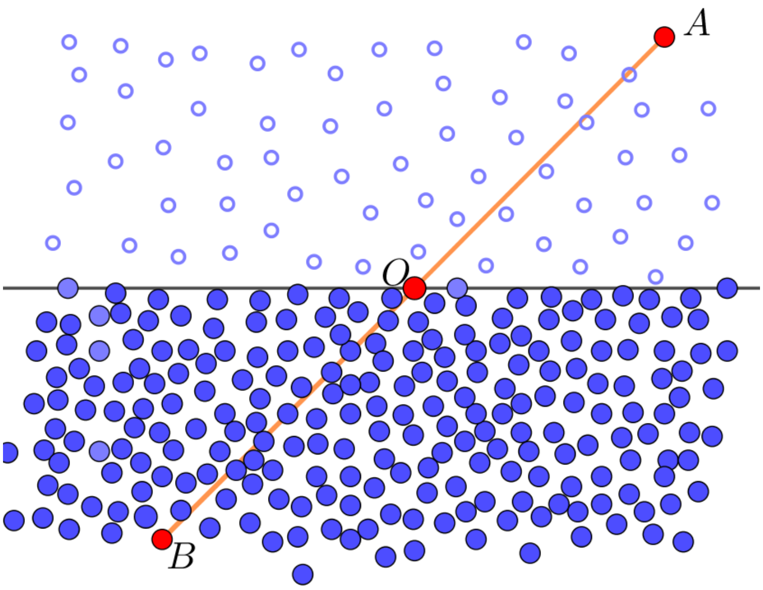
5. Class Exercise:



Plan Your Trip! Normally, the fastest path between two points is a straight line - but not when you hit obstacles, which cause delays.

In the picture here, count the number of dots that touch the path AB to find how much the traveller is slowed down. The top dots represent gas molecules in the air. The lower dots are water molecules.

Now try to plan a better trip from A to B: Choose a point C on the black separating line, connect it to both A and B by straight line segments, and count the total number of dots you crossed. Is it more or less than on the path AB?

6. What causes refraction to happen?

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7. Why do you think the light travels along the fastest paths?

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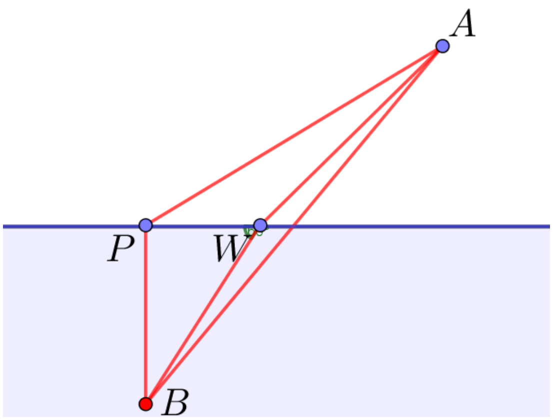
8. In the Light SuperWorld, light rays travel on any paths they like. Three rays called Mr Simples, Mrs Wiseman and James Bold, decide to go from a point A, found 100 meters above water, to a point B, found 100 meters below water. They are warned that travelling through water is slower, namely

- They can travel at a speed of 300 meters/second through air;
- But only 225 meters/second through water.



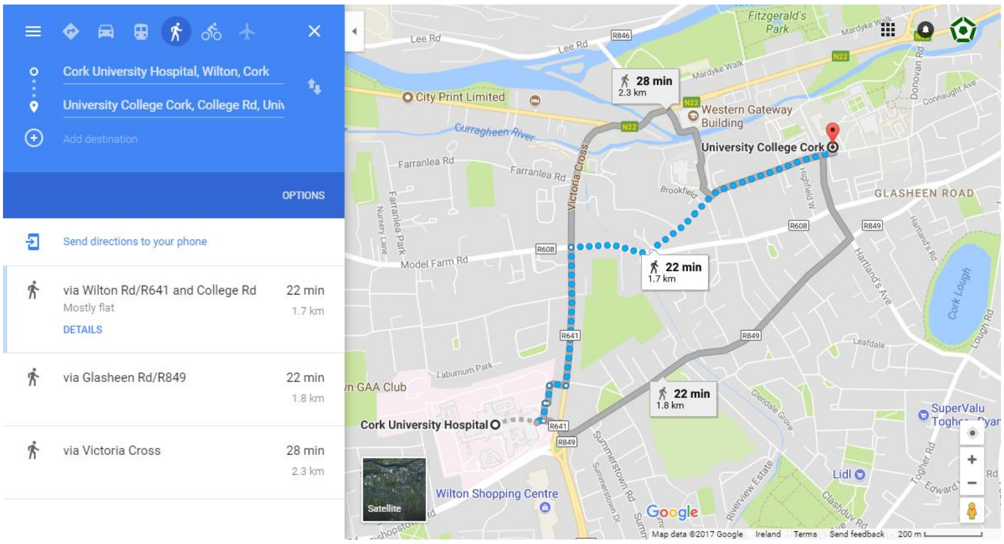
Mr Simples decides to take a straight line from A to B, a total distance of 255 meters.

James Bold decides to go as much as possible through air, so he travels 187 m to point P, found exactly above B on the surface of the water, and then from P straight down to B.

Mrs Wiseman Ray makes some calculation and decides to go about 141.5 meters through the air, heading straight for a point W on the water surface, and then travels about 115.5 meters through the water, from W to B.

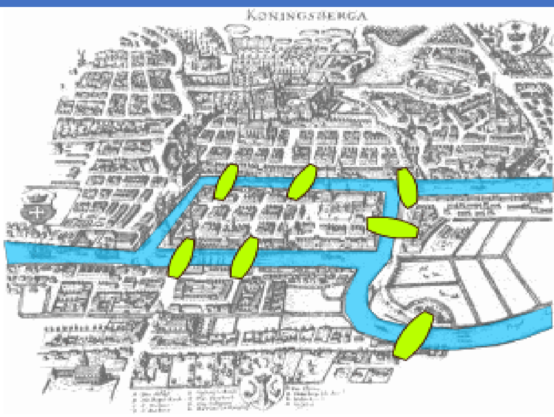
Which Ray gets to the destination fastest? Can you intuitively explain why?

9. Find some things that Google Maps has in common with Hamilton's approach to optics.

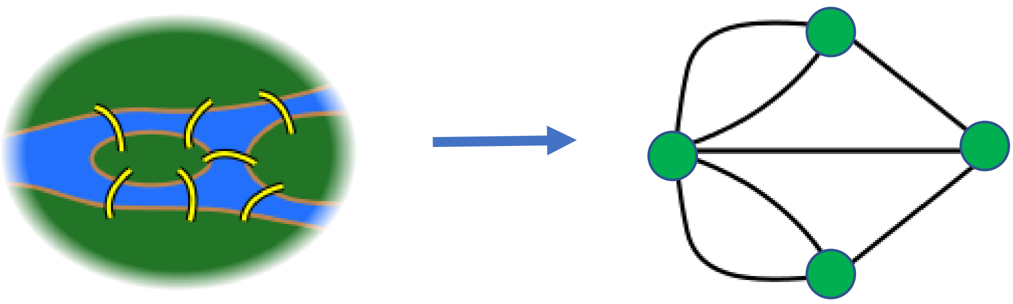


SECTION 3 - GRAPH THEORY

10. What was the game of the Seven Bridges of Königsberg?

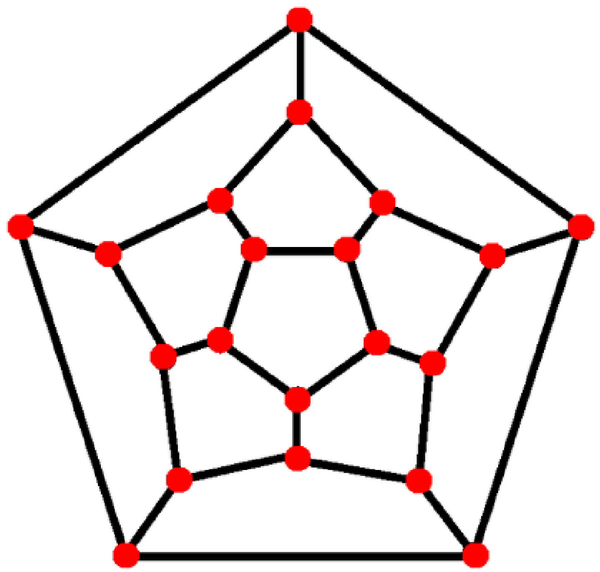


11. Why is it impossible to win the game of the Seven Bridges of Königsberg?



12. What is a graph in graph theory? Draw an example of a graph.

13. What is a Hamiltonian cycle? Find an example on the following graph.



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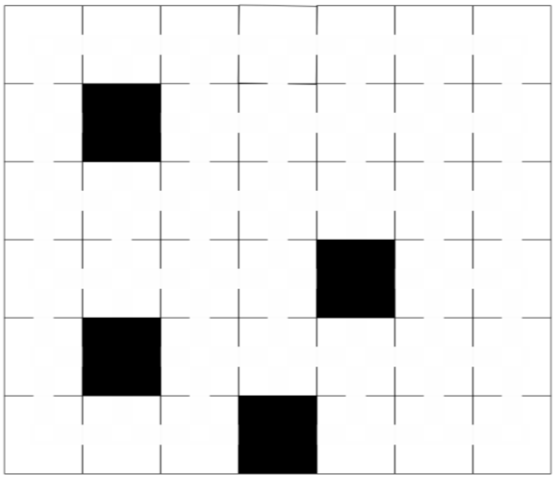
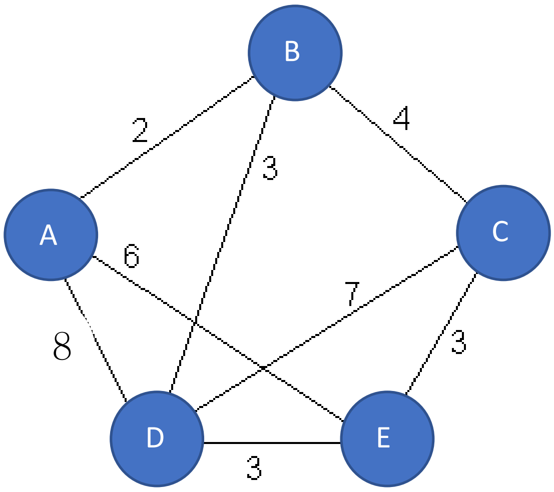
14. Suppose you are a security guard in a museum, and are locking up for the evening. The diagram given here is a plan of the museum, where every square represents a room and darkened squares are rooms that are closed for renovations. Before closing the museum, you have to check each (open) room once and only once and finish back where you started (you can start wherever you like). You must move from room to room and cannot leave the museum. Can you trace out your path in each diagram?



15. Imagine you are a salesperson who travels around a country selling your product in big cities. Some of the cities are linked by highways, while others aren't, and every highway-link between two cities has an associated distance. You want to visit every city exactly once and finish where you started, while at the same time ensuring you travel the smallest distance possible. Which path do you take?



SECTION 4 – ALGEBRA AND GEOMETRY

16. Use reflection to explain why multiplying two negative numbers gives you a positive number: $(-1) \times (-1) = 1$ or $(-1) \times (-2) = 2$.

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17. Take two red lines l and k meeting at O and with an angle $\alpha = 30^\circ$ between them. Reflect a point P across l to get P' and then across k to get P''. If $\angle MON = 30^\circ$ and $\angle POM = 10^\circ$, find $\angle POP''$.

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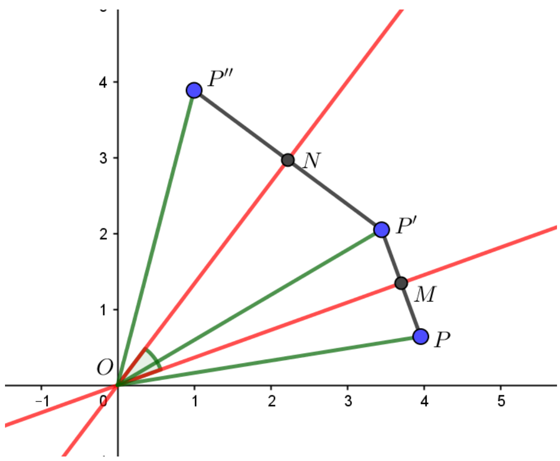

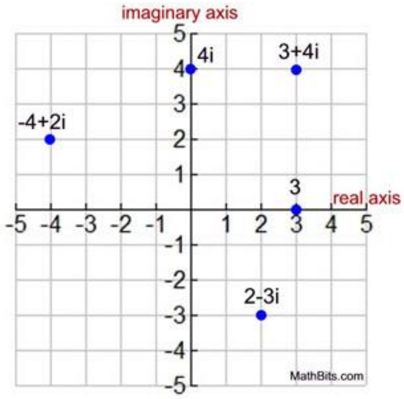

18. Plot the complex numbers $-3+2i$, $3+2i$ and $3-2i$ in the plane to your left. What do you notice about the places they occupy in the plane?

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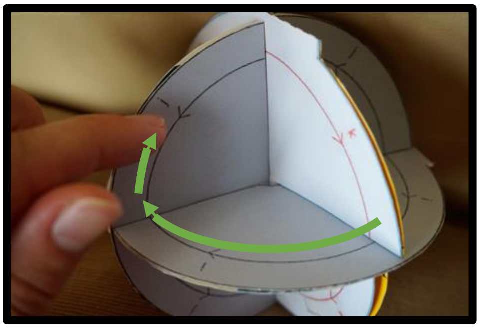


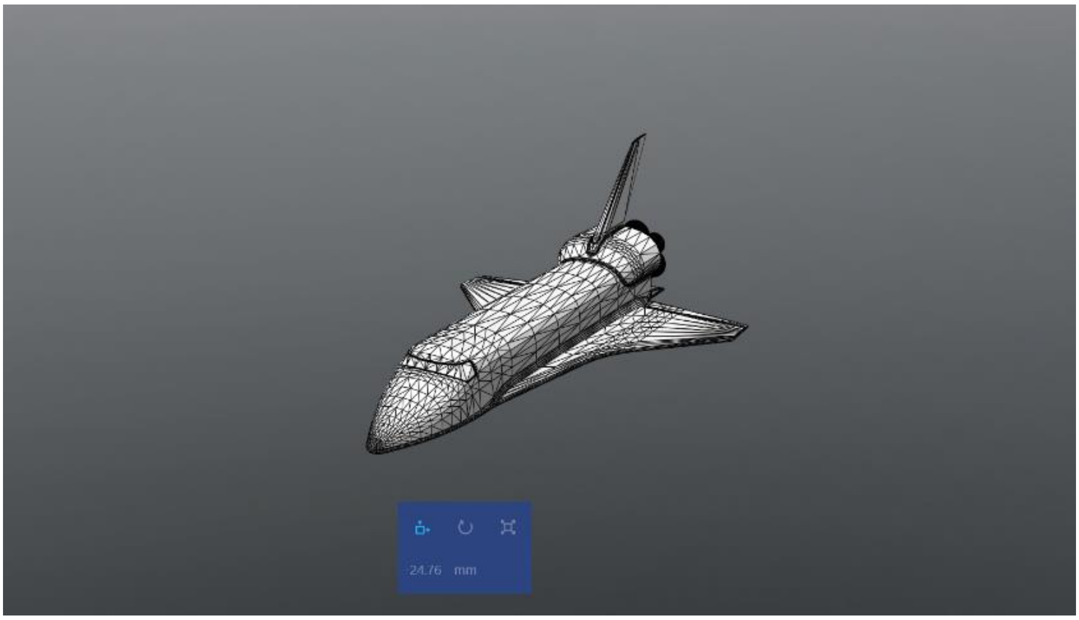
18. We will use the Quaternion Ball to play with quaternion multiplication. As you can see, the Ball is made of three discs that intersect at right angles, with red circles on each disc. We find that tracing a finger along these circles while carrying out this exercise is helpful. Tracing out a quarter circle in the same direction as the arrow on it corresponds to right the quaternion unit (i, j or k) printed next to the circle. Tracing opposite to the arrow's direction corresponds to the negative of the unit. Tracing out a number of quarter arcs in sequence corresponds to each of the units traced out written down next to each other in the same order: to the right, for example, I trace out i and then $-j$, which matches the multiplication $i(-j) = -ij$. Any other path that takes you from the same starting point to the same finish gives an equal answer: here, I could also have taken $-k$ to get to the same point, so I know now that $-ij = -k$, or $ij = k$. Let everyone in the class please copy the table just below onto a sheet of paper, and using the Quaternion Ball as we've described, fill out all the missing entries. Note: Multiplication by 1 does not figure on the Quaternion Ball because it represents staying in place: no change.

Fill in the table with the correct Hamilton products. For each box, its row represents the first number in the product, while its column is the second number.

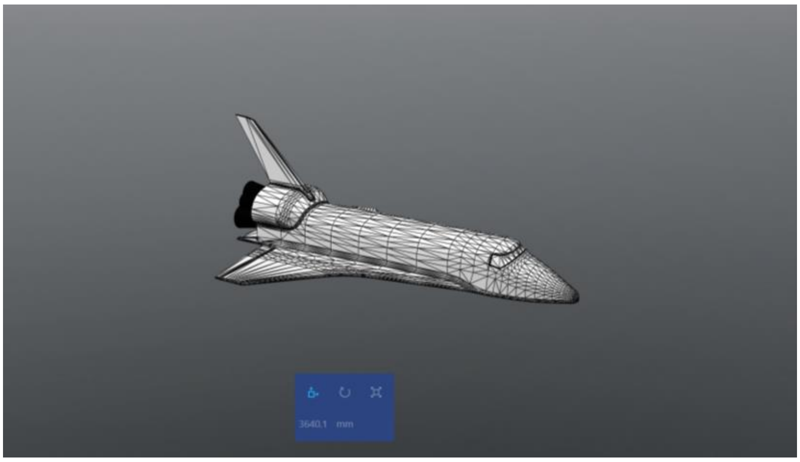
For example, I've placed k in row i and column j because $ij = k$.

$\times$	**1**	***i***	***j***	***k***
1				
i			k	
j				
k				

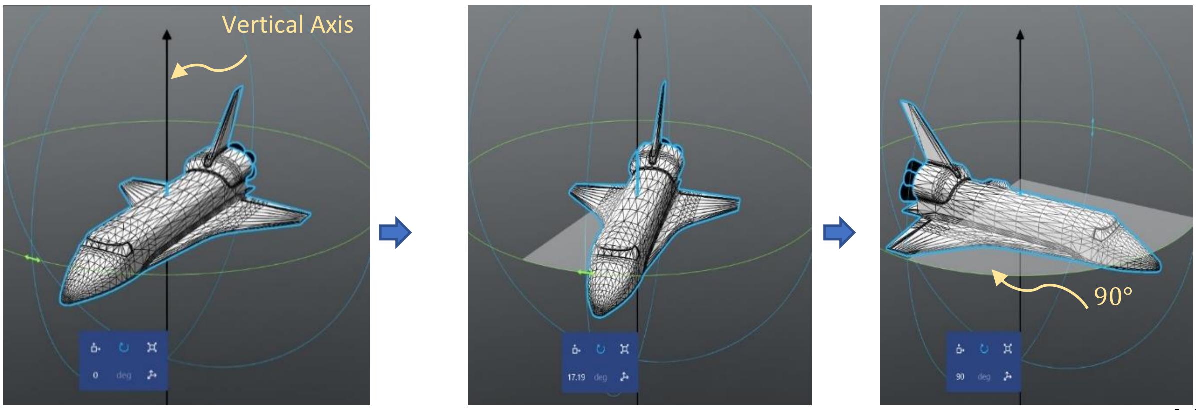
18. Imagine there are astronauts on a spacecraft.



The astronauts are having a tanning competition on board and want to turn the shuttle by 90° to its left so that it faces the sun, like this:



How does the shuttle carry out this command (in terms of quaternions)?



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19. Calculate the products iji, iki and iii.

Now take the number $2i + 3j + k$ and surround it by i and i, like this: $i(2i + 3j + k)i$. Plot your results in 3D space. Try to describe in words what happened to $p = 2i + 3j + k$ when you surrounded it by i-s.